3D TRANSFORMATIONS

- **HOMOGENOUS COORDS. [x y z h]**
- **TRANSLATION**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & 1 \end{bmatrix}$$

- **ROTATION ABOUT X**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **ABOUT Y**

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

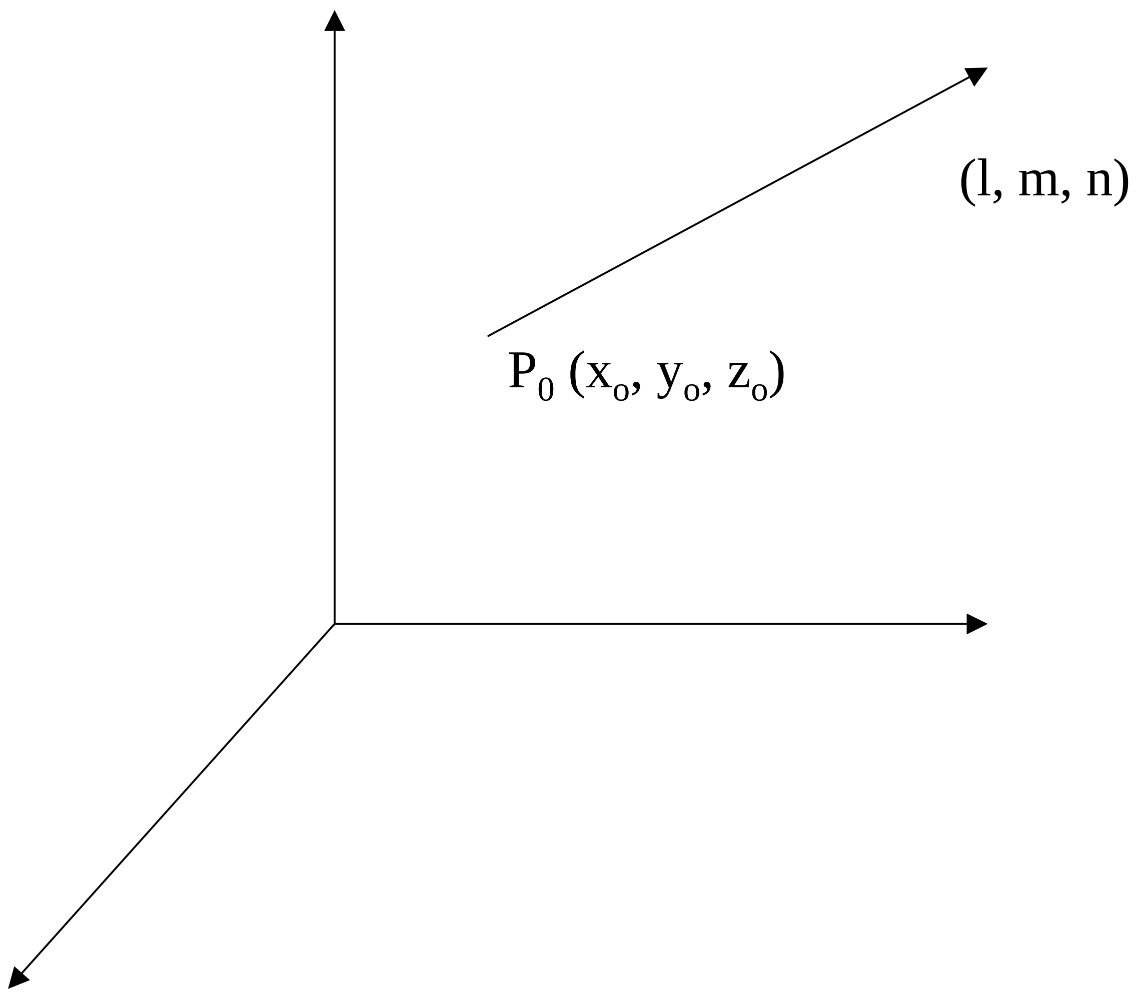
- **ABOUT Z**

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **SCALING**

$$\begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & S \end{bmatrix}$$

GENERAL 3D ROTATION



STEPS

1. TRANSLATE ORIGIN TO P_0

2. ALIGN VECTOR WITH AXES (say, Z)

   i. ROTATE TO BRING VECTOR IN XZ PLANE
   ii. ROTATE TO BRING VECTOR ALONG Z-axis

3. ROTATE ABOUT Z-AXIS

4. REVERSE STEPS 2

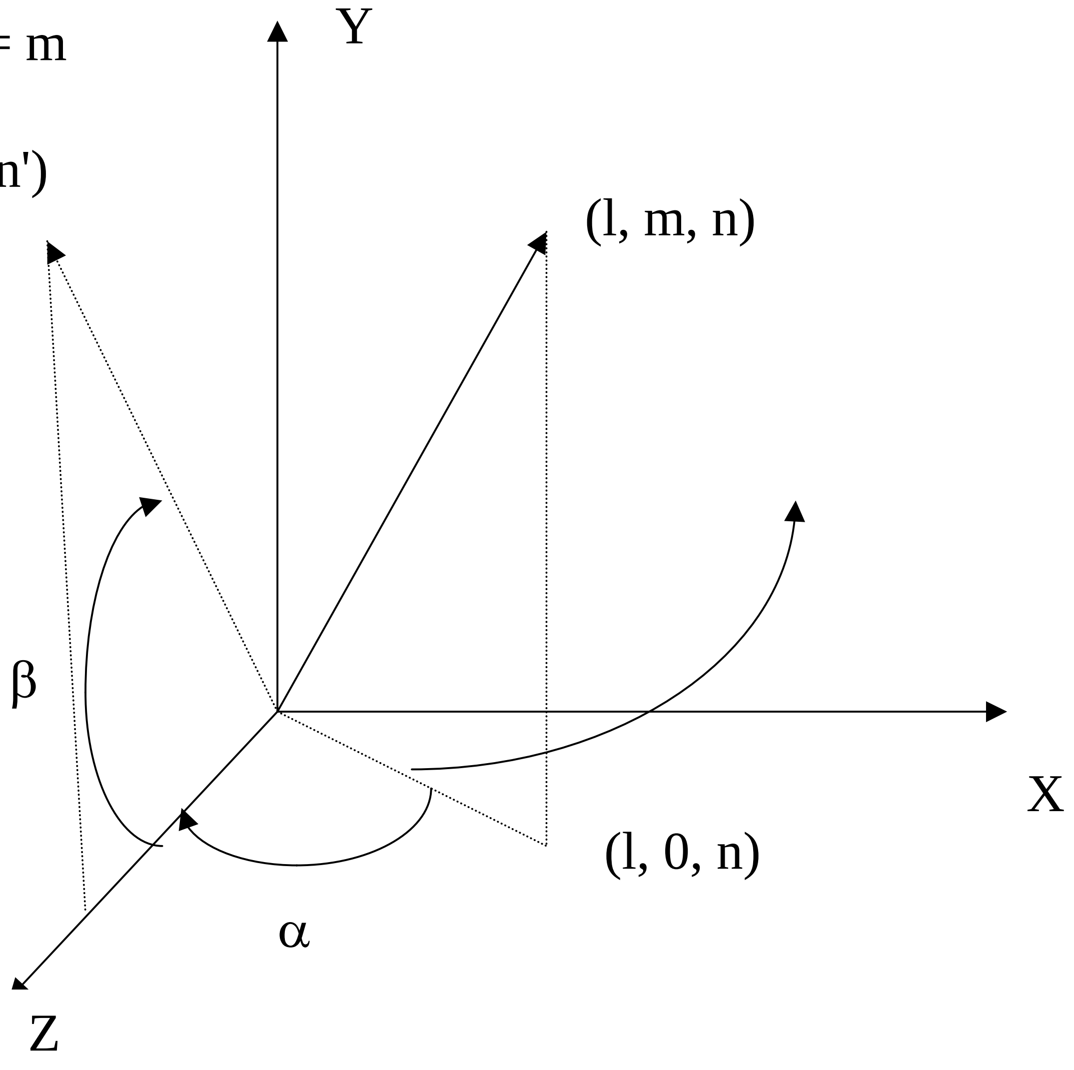
5. REVERSE STEP 1

$\cos \beta = n'$

$\sin \beta = m$

$n' = \lambda$

$(0, m, n')$



- **STEP 2-i:**

Rotate Axis by α CCW about Y

- **STEP 2-ii.**

Rotate Axis by β CCW about X

DEVELOP TRANSFORMATION MATRICES

FINAL ROTATION MATRIX

$$R \equiv \begin{bmatrix} r11 & r12 & r13 & 0 \\ r21 & r22 & r23 & 0 \\ r31 & r32 & r33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **TOP LEFT 3 X 3 MATRIX IS ORTHOGONAL ($AA^T = I$)**
- **COLUMNS OF THIS MATRIX ARE TRANSFORMED BY R TO GIVE PRINCIPLE AXIS**

$$P \equiv \begin{bmatrix} r11 & r21 & r31 & 1 \\ r12 & r22 & r32 & 1 \\ r13 & r23 & r33 & 1 \end{bmatrix}$$

- **PR = I; HENCE TRANSFORMATION R TRANSFORMS AXIS TO P.**

- **CONSIDER LOCAL COOD-SYS DEFINED BY THE ROTATION AXIS**

  $$u_z' = u = (l\ m\ n) = (u_{z1}'\ u_{z2}'\ u_{z3}')$$

  $$u_y' = u \times u_x / |u \times u_x| = (u_{y1}'\ u_{y2}'\ u_{y3}')$$

  $$u_x' = u_y' \times u_z' = (u_{x1}'\ u_{x2}'\ u_{x3}')$$

  (ORTHONORMAL BASIS)

- **DEFINE**

$$R \equiv \begin{bmatrix} ux1' & uy1' & uz1' & 0 \\ ux2' & uy2' & uz2' & 0 \\ ux3' & uy3' & uz3' & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **R TRANSFORMS AXIS TO ALIGN Z WITH u (THRO' ORIGIN)**

- **HENCE**

TRANSLATE ORIGIN TO P0
PERFORM R
ROTATE BY θ
REVERSE R
REVERSE TRANSLATION

OTHER TRANSFORMATIONS

- **SCALING --------??**

- **REFLECTIONS**

  X-AXIS: $S_Y = S_Z = -1; S_X = S = 1$

  XY-PLANE: $S_Z = -1; S_X = S_Y = S = 1$

  ORIGIN: $S_X = S_Y = S_Z = -1; S = 1$

- **SHEAR (Z CONST, X,Y CHANGE PROP TO Z)**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **SCALING wrt A GIVEN POINT**

  TRANSLATE ORIGIN TO POINT
  SCALE
  TRANSLATE BACK

- **REFLECTION ABOUT A PLANE**

  TRANSLATE ORIGIN TO POINT ON PLANE
  ALIGN PLANE NORMAL WITH AXIS REFLECT
  REVERSE EARLIER TRANSFORMATIONS